What is Electromagnetism?

Let's first ask ourselves a fundamental question, "What *is* Electromagnetism?"

In physics, there are four fundamental laws of nature that govern all physical phenomena in the universe and electromagnetism is one of these four forces. Those four physical phenomena are:

Gravitational Force (Gravity) - Pulls masses together including planets, stars, and black holes. Also governs orbiting bodies, expanding the universe, and falling apples.

Strong Nuclear Force (Atoms) - Binds protons and neutrons in the atomic nucleus.

Weak Nuclear Force (Electrons) - Responsible for radioactive decay and allows particles to change types. Essential for nuclear fusion.

<u>Electromagnetic Force</u> (Fields) - Acts on charged particles; responsible for electricity, magnetism, and light

At its core, electromagnetism describes how electrically charged particles interact with each other as well as electric and magnetic fields. It's a force. It's not a very strong force physically, but it's the cornerstone of how we communicate over the phone, how we are able to peer into the universe to see galaxies and stars, how your computer works, it's used in MRI's for verifying cancerous tumors, and how your debit card works. Needless to say, RF is used everywhere, all the time, all day, every day.

I mentioned the word field, so what exactly is a field? A field is a physical quantity that has a value at every point in space and time (or a value and a direction - a vector). To picture a field in everyday life, imagine a massive room at roughly 77 degrees Fahrenheit. I can walk to the corner of this room and it might be 75 degrees and I can walk to the other corner at the end of the room and it might be around 78 degrees. At every point in this space and time, there is a specific temperature point/value. Electromagnetism, like heat, is a field - each with its own physical "value" at each point in space and time where the field exists.

Translating this to electromagnetism, an electric field shows the force that would act on a positive charge at any point in space and time. A magnetic field on the other hand, describes how magnetic forces are distributed in space. An electromagnetic field combines electric and magnetic fields into one unified field.

The Goal for Electromagnetic Theory in RF

The goal, specifically for electromagnetism in RF Engineering, is to show that electric and magnetic field lines can travel as waves through space. It's been established that it's a force, but can this force travel over distance? These questions lay the foundation for energy propagation (travel) for all wavelengths on the electromagnetic spectrum (light, radio waves, X-Rays and microwaves). The mathematical description of electric and magnetic field lines propagating through space is called the wave equation. From the wave equation, it is possible to derive the simplest type of electric/magnetic wave propagating through free space - a plane wave.

Why is all this important? Plane waves are a simplified, but powerful building block for understanding how electromagnetic waves behave in bounded structure media, like waveguides or transmission lines (ways of guiding electromagnetic waves through a structure). Think of plane waves as the most fundamental electromagnetic wave found in nature - very useful for understanding wave propagation without complicating the phenomenon right out the gate.

Guided waves on the other hand are similar, but instead give a superposition of plane waves and can be formally expressed as combinations of many plane waves reflecting off the guide boundaries at specific angles. What is superposition? In terms of plane waves, superposition is the idea that when two or more plane waves overlap, their combined effect is simply the sum of their individual amplitudes at any point. To illustrate, imagine you're on a lake enjoying the beautiful weather and two boats drive by each other in opposite directions. Both of the boats will produce their own individual waves. Once the waves from each boat reach each other, they will combine to create a singular wave - the sum of the waves from the first and second boat. Sometimes a big splash if the peaks line up, or a calmer patch if they cancel out. In the real world, there are *tons* of plane waves interacting with each other simultaneously. The easiest way to accurately describe these interactions is with superposition.

This superposition provides a mode composition of complex field distributions such as Transverse Electric (TE), Transverse Magnetic (TM), or Transverse Electromagnetic (TEM) waves. This is particularly important because in most transmission lines, the dominant mode of wave propagation is TEM - meaning a TEM wave is a uniform plane wave confined between two conductors under the ideal TEM/quasi-static approximation. More specifically, the voltage and current on a transmission line are directly related to the electric and magnetic fields of the TEM wave. This is crucial, because voltage and current wave propagation can be simplified into a circuit model - giving way to topics such as impedance and reflection on a transmitted voltage/current wave on a transmission line.

The reason why we can design and simplify using circuit models **<u>is because of electromagnetic wave propagation.</u>** Before we can do anything, we need to start with Maxwell's Equations and make six underlying assumptions to mathematically prove these waves can propagate. Only then, we can start understanding circuit models and really harness the power of radio frequency circuits.

Now that we've built some intuition for what RF waves look like and why frequency even matters, it's time to peel back the curtain on the real backbone of electromagnetics. Every RF phenomenon you'll every see - from antennas radiating to microstrip launching modes to a simple trace on a PCB carrying a GHz signal - comes from four equations.

Yep. Four.

If you want to understand RF at a level that actually lets you troubleshoot confidently, you need to at least see where these equations lead. We're going to step into the math, but I'll walk through it in a way that connects each step back to something physical and intuitive.

Before we jump into Maxwell's equations and the mathematics behind electromagnetic fields, there are four vector calculus tools that appear frequently in electromagnetics. They look intimidating at first, but each one has a simple physical meaning. You don't need to hold a degree in mathematics to understand them - you only need intuition.

Divergence - ($\nabla \cdot E$ and $\nabla \cdot B$)

Divergence tells you whether a field is spreading out or converging at a point. For the electric field, $\nabla \cdot E = \frac{\rho}{\epsilon_o}$, simply means *charge* is the source of the electric field lines. Positive charges push the lines outward and negative charges pull them inwards. For magnetic fields, $\nabla \cdot B = 0$, is nature's way of saying magnetic field lines never begin or end - they always loop. No magnetic monopoles exist.

Curl - ($\nabla \times E$ and $\nabla \times B$)

Curl measures how much a field wants to swirl or circulate around a point. Faraday's Law, $\nabla \times E = -\frac{\partial B}{\partial t}$, tells you that a changing magnetic field generates a curling electric field. Ampere - Maxwell's Law, $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$, says currents and changing electric fields create a curling magnetic field. This back and forth "twisting" behavior is what makes electromagnetic waves self-sustaining: a changing electric field curls up a magnetic field and a changing magnetic field curls up an electric field

Laplacian - ($\nabla^2 E$, $\nabla^2 B$)

The Laplacian ($\nabla^2 E$, $\nabla^2 B$) compares a field at one point to the field around it - it measures *curvature*. Where the field bends or differs from its surroundings, the Laplacian captures that spatial variation. When combined with the time derivatives in Maxwell's equations, it forms the wave equation, showing how the electric and magnetic fields propagate and smooth themselves out through space.

The Del Operator - (∇)

The del operator is the spatial derivative toolbox behind every form of Maxwell's equations. Apply $\nabla \cdot$ and you get divergence (sources). Apply $\nabla \times$ and you get the curl (circulation). Combine ∇ with itself (∇^2) and you get curvature and the wave equation.

Everything in electromagnetic field behavior is encoded in how ∇ interacts with *E, H,* ρ, and *J.*

1.1 Maxwell's Equations

Maxwell initially developed 12 equations to explain electromagnetic fields and waves, but was later simplified to four thanks to Heaviside (another shoutout to Oliver). These equations can be utilized in its differential and integral form.

Equation 1. Gauss's Law for Electricity

$$\nabla \cdot E = \frac{\rho}{\epsilon_o} \tag{1.1.01}$$

Equation 2. Gauss's Law for Magnetism

$$\nabla \cdot B = 0 \tag{1.1.02}$$

Equation 3. Faraday's Law of Induction

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1.1.03}$$

Equation 4. Ampere's Law with Maxwell's Addition

$$\nabla \times B = \mu_0 \ J \ + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \tag{1.1.04}$$

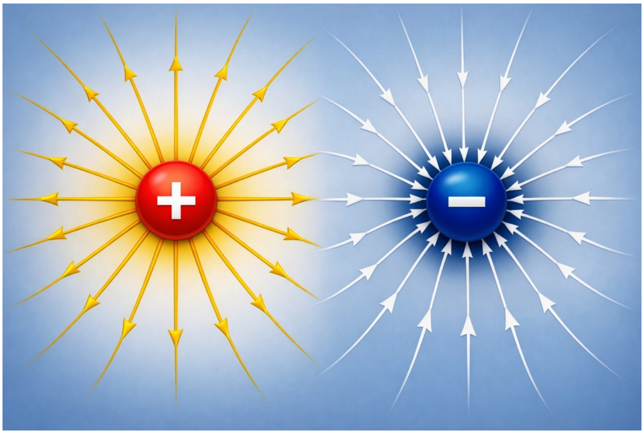
So, what are these four equations? What do they describe? We know it describes something over a region of space and time, but what exactly?

Gauss's Law for Electricity - Sources Create Fields

Gauss's Law for Electricity states that electric field lines diverge from electric charges and the divergence of the electric field E at a point is proportional to the charge density at that point. Positive charges are sources of electric fields and negative charges are considered "sinks".

$$\nabla \cdot E = \frac{\rho}{\epsilon_o}$$

Electric field lines are invisible "forces" that come out of electric charges - whether that be a positive or a negative charge. A positive charge can be thought of as a sun - positive charges push electric field lines outward. Negative charges can be thought of as whirlpools in a body of water - they pull electric field lines inward. Gauss's Law for Electricity measures how much charge is inside of a closed space by looking at the electric field lines passing through the surface around it. It tells how much charge is inside of a closed space by measuring how much of the electric field is flowing through the closed space.



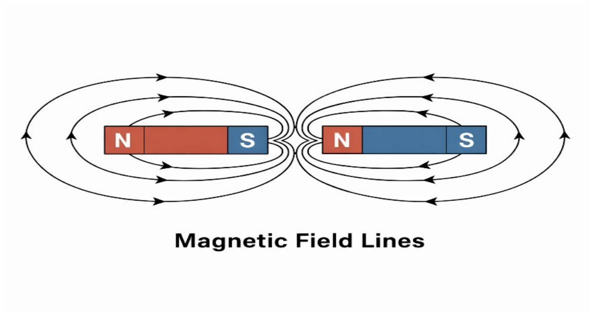
Positive and Negative Charges

Gauss's Law for Magnetism (No Magnetic Charges Exist)

Gauss's Law for Magnetism states there are no magnetic monopoles. Magnetic field lines are always closed loops. The divergence being zero means there's no net 'magnetic charge' anywhere.

$$\nabla \cdot B = 0$$

Electric fields can start or end on electric charges whether that's positive or negative. Magnetic fields can't. Magnetic field lines make loops - there is no stop or end unlike electric field lines.



Magnetic Field Lines

Essentially, there's no such thing as a magnetic charge (magnetic monopole). Very similar to a sphere, there's no "start" or "end".

In other words, you can't have a north pole without a south pole. Just try cutting a magnet in half. You will get two smaller magnets each with its own north and south pole. So, if the magnetic field lines are always a loop with no start or finish, the net magnetic field going into the bubble vs out of the bubble is zero. Magnetic lines do not start or stop - they just loop around.

Faraday's Law of Induction (Changing Magnetic Fields Create Electric Fields)

The curl of the electric field, E, describes how the electric field 'loops' around, and this curling is caused by the time rate of change of the magnetic field, $\frac{\partial B}{\partial t}$. This is the basis of how generators and transformers work. A changing magnetic field creates a looping electric field. If a magnetic field changes over time, it induces an electric field that forms a loop - it doesn't just point outward like from a charge. This means the electric fields can curl even when there are no electric charges around - but only if the magnetic field is changing.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

To simplify further, if the magnetic field is changing in any form, the magnetic field will twist up an electric field around it - sort of like stirring up a whirlpool in water.

Ampere's Law (with Maxwell's Addition) - Currents and Changing Electric Fields Create Magnetic Fields

A magnetic field curls around electric currents (denoted as J) and also around changing electric fields (Maxwell's addition). The first term is the original Ampere's law, $\mu_0 \ J$, where current generates magnetic fields. The second term, $\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$, accounts for displacement current, which is necessary to preserve continuity in situations like charging a capacitor - a changing electric field also creates magnetic fields.

$$\nabla \times B = \mu_0 \ J \ + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Let's combine all four of these laws and make a couple of assumptions to achieve our goal - deriving electromagnetic wave propagation in free space.

To get something clean out of Maxwell's equations - something we can actually solve - we need to simplify the physical world just a bit. Not because the real world is simple, but because peeling back the complexity lets us see the underlying structure. These six assumptions aren't random. They're the mathematical "setup" that makes it possible to derive the most fundamental type of electromagnetic wave: the plane wave.

The Six Assumptions

To mathematically prove electromagnetic fields can travel through space, six assumptions need to be made. Think of them as initial conditions to prove these waves can propagate through space.

1. The permittivity, ϵ, is isotropic (does not vary in space)
2. There is no free movable charge.
3. The permeability, μ, is isotropic (does not vary in space)
4. The material is not conductive
5. The time harmonic variation of the electric field is sinusoidal
6. The wave changes sinusoidally in the z direction and has a constant value everywhere in x and y direction

Let's break down each of these assumptions further.

Assumption 1 - The permittivity, ϵ, is isotropic (does not vary in space)

Permittivity is constant everywhere. It does not change or vary from one point to another.

Permittivity is a measure of how well **electric** field energy can be stored in a confined area. Higher permittivity means more electric field energy can be stored in a confined space and a lower permittivity means less electric field energy can be stored in a confined space. In this assumption, every point in space can hold the same amount of electric field energy.

Assumption 2 - There is no free movable charge (No free charge density)

There are no charges moving around (positive or negative). No sources of charge density exist. No suns or whirlpools.

Assumption 3 - The permeability, μ, is isotropic (does not vary in space)

Permeability does not change or vary from one point to another. Unlike permittivity, permeability is a measure of how well *magnetic* field energy can be stored in a confined area. Higher permeability means more magnetic field energy can be stored in a confined area and less permeability means not as much magnetic field energy can be stored in a confined area.

Assumption 4 - The material is not conductive

This assumption states that the medium the wave is traveling in does not conduct electricity. No charge (positive or negative) can travel from one point to another via electromotive forces.

Assumption 5 - The time harmonic variation of the electric field is sinusoidal

A time harmonic sinusoidal field states that the field varies sinusoidally with time at a single angular frequency. If you're standing at a fixed point in space and watching the electric field, it will oscillate sinusoidally over time - meaning the field is vibrating back and forth much like a wave on a guitar string. In this case, the magnitude and direction of the electric field is oscillating.

Assumption 6 - The wave changes sinusoidally in the z direction and has a constant value everywhere in x and y direction

This assumption is really two in one. The first part of this assumption states the wave changes sinusoidally in the z-direction. This sinusoidal change describes a traveling wave moving in the positive or negative z direction. The second part of this assumption states the wave has a constant value everywhere in the x and y direction. This implies planar wavefronts that are perpendicular to the direction of propagation. This means all points in a given x-y plane "feel" the same field strength and phase at a given moment in time.

Think of this wave as multiple sheets of paper standing upright and moving in a singular direction - a plane if you will. In free space (a vacuum) EM waves naturally form *plane wave* solutions.

After assuming time-harmonic sinusoidal conditions (like the one pictured above), these signals can be analyzed at a specific angular frequency as mentioned before.

Deriving the Wave Equation/Helmholtz Solution for the Electric Field

Let's simplify Maxwell's Equations using what we know so far from the six assumptions.

Assumption 1: Since the permittivity is a scalar constant and isotropic, ϵ, can be moved outside derivatives.

Assumption 2: Since there are no free movable charges, $\rho = 0$. This means we can simplify Gauss's Law for Electricity. There's no charges moving through the bubble, so the net electric field moving through this space is zero.

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot E = 0$$

Assumption 3: Since the permeability is a scalar constant and isotropic, μ, can be moved outside derivatives.

Assumption 4: The medium is non conductive. This means $J = 0$. If the medium does not conduct electricity, there's no conduction current. This means we can simplify Ampere's Law (with Maxwell's Addition).

$$\nabla \times B = \mu_0 \; J \; + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$
$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Rewriting the Equations Applying Assumptions 1 - 4:

1. Gauss's Law for Electricity

$\nabla \cdot E = 0$

2. Gauss's Law for Magnetism

$\nabla \cdot B = 0$

3. Faraday's Law of Induction

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

4. Ampere’s Law (no currents)

$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Now that we have stated our initial conditions and have simplified the equations as much as possible. Let’s mathematically prove electromagnetic waves can travel through space and time.

Step 1: Take the curl of Faraday’s Law:

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = -(\frac{\partial B}{\partial t} \times \nabla)$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$$

Step 2: Substitute Ampere’s Law into the above equation:

$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\mu_0 \epsilon_0 \frac{\partial E}{\partial t})$$

Step 3: Extract the permittivity and permeability scalar constants and combine partial derivatives.

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\nabla \times (\nabla \times E) = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

What we're doing here may look like a lot of vector calculus, but there's a payoff coming. Every step is stripping away what we don't need so that we can reveal the simplest, cleanest form of an electromagnetic wave. If Maxwell's equations are the full engine, the Helmholtz equation is the "pure note" that comes out of it - the part that tells us how waves actually move through space.