5.1 Network Analysis

Network analysis is a fundamental tool in radio frequency (RF) engineering used to simplify the study of complex linear networks, as well as more basic structures. For example, a small RF component such as a directional coupler may have up to four ports. But what exactly is a *port*?

A port is a defined point where RF signals either enter or exit a device. While the internal structure of an RF component may involve complex distributions of voltage and current, network analysis abstracts this complexity by focusing on the incident and reflected waves at the ports themselves.

In more elaborate systems - such as a device containing multiple couplers and interconnects - there could be dozens of ports and internal connections. Analyzing each sub-component individually would be both time-consuming and cumbersome. Network analysis streamlines this process by allowing engineers to treat the entire system in terms of its port behavior, described using S-parameters. This approach enables rapid characterization and efficient design of even highly complex RF systems.

But what exactly is network analysis? I mentioned earlier how network analysis analyzes the incident and reflected waves from ports, but how? We first need to start with wrapping our brains around the fact that there is this black box of any number of RF devices for any network. This is one of my favorite analogies for network analysis from my RF professor, Dr. Burke.

5.2 ASLOTI

Let's imagine one of the most common networks out there - a 2 Port network.

A two-port network has an input (Port 1) and an output (Port 2). When you send an RF signal into Port 1, part of it travels through the circuit toward Port 2, and part of it might bounce back toward the source. We call the wave going into Port 1 a_1, the wave coming out at Port 2 b_2, the reflection back from Port 1 b_1, and any wave entering from Port 2 a_2. In simple terms, "a" waves go into the circuit, and "b" waves come out of it. You can think of this like water flowing through a pipe: you pour water in at one end (a_1), most of it flows out the other end (b_2), but if there's a blockage, some splashes back toward you (b_1). We measure these incident and reflected waves to understand how well energy moves through the circuit. If too much signal bounces back (like water splashing), it means poor impedance matching, which can cause signal loss and inefficiency. The goal is to maximize the forward signal and minimize reflections for the best performance.

Inside this box, there can be many different devices that are used as part of the overall device. In passive networks (just metal - nothing else), there could potentially be devices such as power dividers, combiners, hybrids, filters, and directional couplers. In active networks, there can be devices such as amplifiers, transistors, oscillators, and frequency sources. Dr. Burke, my undergraduate professor for my RF courses at WNEU, likes to slap an acronym over this box. Inside this box there's ASLOTI. In other words, there's **A Shit Load Of Things Inside** the box that makes up the circuit between ports (Port 1 and Port 2 in this scenario). For large networks, it can be multiple devices as mentioned before, but for singular device networks such as a filter, it can be various impedances and/or distributed elements, complex voltages, and currents. There's ASLOTI.

5.3 Scattering Parameters

The main type of network analysis we will focus on is based on scattering parameters, or S-parameters. These parameters describe how signals behave when they encounter changes in an RF network - specifically, how much of a signal is reflected back and how much is transmitted through. Instead of measuring voltages and currents directly, S-parameters deal with incident and reflected power waves, which makes them ideal for high-frequency circuits.

Scattering parameters can describe networks with any number of ports, commonly referred to as N-port networks. Most RF devices use one to four ports. For example:

1-port components such as 50 Ohm terminations
2-port components such as filters, and attenuators
3 or 4 port components such as couplers or power dividers.

The total number of S-parameters for a network is equal to N^2 because each port can both send and receive energy from every other port, including itself. For example, a 1-port network has one S-parameter S_{11}, a 2-port network has four (S_{11}, S_{12}, S_{21}, S_{22}), and a 3-port network has nine, and so on.

5.4 Kurokawa's Power Waves

Unlike parameters such as Z, Y, or ABCD, S-parameters do not require ports to be terminated in a specific way (such as open, short, or perfectly matched) to analyze the network. This flexibility makes them especially practical for high-frequency design.To define these waves formally, we will use Kurokawa's power-wave definitions.

Let's pump the brakes. I personally think it's always important to highlight the men or women behind the work that changed an entire field for decades and Kurokawa is no exception. A little bit about the man himself - Kurokawa was a Japanese electrical engineer who attended the University of Tokyo in the middle to late 1950's where he earned his BSEE and PhD in Electrical Engineering. After serving some years as a professor at the university, Kurokawa went to work for the legendary Bell Laboratories from 1963-1975. During his tenure at Bell Labs, Kurokawa made significant advancements in microwave technology, including the development of microwave balanced transistor amplifiers and millimeter-wave modulators. He also played a pivotal role in the early development of optical fiber systems, but none of that work would compare to his 1965 paper, "*Power Waves and the Scattering Matrix*". In the long and short of it, Kurokawa provided a more accurate method for analyzing power flow in microwave networks, addressing limitations of previous models that assumed real characteristic impedances. Before Kurokawa, RF/microwave engineers were fine with using voltage-wave-based scattering theory. For the time, it was "good enough" for real-world systems, but unfortunately, in real-world systems, the characteristic impedance of the circuit isn't purely real. Instead of conforming to the "good enough" standard, Kurokawa found another way. A better way. Truly a revolutionary in microwave engineering and a legend in the field to say the least.

At high frequencies, traditional voltage and current waves can be problematic for power analysis in mismatched systems, especially at high frequencies. Kurokawa wanted to create a system of equations where power conservation is satisfied and reflection coefficients remain meaningful in complex impedance environments. In classic transmission line theory, the overall voltage on the line is expressed in forward (incident) voltage/current waves, reflected voltage/current waves, and a propagation constant.

$$V(z) = V_o^{+} e^{-j\beta z} + V_o e^{+j\beta z}$$

$$I(z) = \frac{V_o^{+}}{Z_o} e^{-j\beta z} - \frac{V_o^{-}}{Z_o} e^{+j\beta z}$$

Where the reflection coefficient is:

$$\Gamma_L = \frac{V_o^{-}}{V_o^{+}}$$

This is great if the reference impedance, Z_o, is purely real (i.e. if the line is lossless), but if the reference impedance is complex, power is not simply proportional to $|V|^2$. This can lead to some confusion about how much power is delivered vs. reflected. Voltage and current waves remain valid, but do not preserve power normalization when the reference impedance is complex.

This is where Kurokawa's Power Wave Definitions come into play. Kurokawa waves allow consistent power calculations even when Z_o is complex or for non-reciprocal, lossy systems. Kurokawa introduced power-normalized waves so that:

$$|a|^2 = incident\quad power\,|b|^2 = reflected \vee transmitted\quad power$$

More specifically,

$$a = \frac{V + I Z_o}{(2\sqrt{\Re(Z_0)})} \quad (5.4.01)$$

$$b = \frac{V - I Z_o^*}{(2\sqrt{\Re(Z_0)})} \quad (5.4.02)$$

All voltage and currents are evaluated at the port reference plane.

Breaking apart these equations:

a is the incident power wave amplitude

b is the reflected or transmitted power wave amplitude

Units: $\sqrt{W}$ (power wave amplitude)

Z_o^* is the complex conjugate of the reference impedance (important if Z_o is not purely real)

So, the power delivered to the port is:

$$P = |a|^2 - |b|^2 \quad (5.4.03)$$

If there's no reflection ($b = 0$), all the incident power is delivered by the load.

Why is this important? Well, it ensures that when S-parameters are used, they describe the ratio of reflected/transmitted to incident power waves, which is physically meaningful. As a result, S-parameters are ratios of wave amplitudes rather than direct power ratios (which is why logarithmic magnitudes use a factor of 20 instead of 10)